Control of Piecewise-Affine Hybrid Systems

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Motivation for control of hybrid systems
In engineering, control computers are used to implement controllers. Interaction of continuous and of discrete dynamics is too tight for control synthesis to ignore.

Examples

2. Conveyor belts (TUE, CWI).
3. Chemical plants (TUE, CWI).
Outline

1. Introduction
2. Concepts
3. Problems and approaches
4. More concepts
5. Control synthesis ASP - Control to facet
6. Control synthesis PAHS
7. Concluding remarks
**Def. Piecewise-affine hybrid system** (PAHS, CT, Time-invariant)

- $Q$ finite **state set**, $U \subset \mathbb{R}^m$, $Y \subset \mathbb{R}^p$, polyhedral sets,
- $X_q \subset \mathbb{R}^{n_q}$, $\forall q \in Q$, closed polyhedral sets,
- $\dot{x}_q(t) = A(q)x_q(t) + B(q)u(t) + a(q)$, $x_q(t_0) = x_q^+$,
- $y(t) = C(q)x_q(t) + D(q)u(t) + c(q)$,
- $e \in E_{in}$, **input event**, or
- $e \in E_{cd}$, if $x(t_1) \in G_q(e) \subset \partial X_q$, **guard**;
  - event generated by continuous dynamics; then **transition**,
- $q^+ = f(q^-, x_{q^-}, e)$, $q_0$,
- $x_{q^+}^+ = A_r(q^-, e, q^+)x_{q^-}^- + b_r(q^-, e, q^+)$, **reset map**.

**Assumptions** (1) Finite number of events at any time.
(2) Finite number of events on any finite interval (non-Zenoness).
Remarks on choice of PAHS

- Examples mentioned belong to class PAHS.
- In hybrid systems, modeling of events generated by continuous dynamics. Event occurs when boundary of state set is reached.
- Polytope as state set, defined by a finite set of inequalities.
- PAHS generalizes timed automata and automata on rectangles analyzed by computer scientists.
- PAHS class of hybrid systems proposed by E.D. Sontag.
- PAHS related to:
  - Mixed Logic Dynamical Systems [A. Bemporad, M. Morari] (DT only) and to
  - Linear Complementarity Systems [M. Heemels].
- System theory - Choice of class of dynamic systems.
**Def.** Affine system on a polytope. (FDAPS)

\[
\begin{align*}
\dot{x}(t) & = Ax(t) + Bu(t) + a, \quad x(t_0) = x_0 \in X_0 \subseteq X, \\
y(t) & = Cx(t) + Du(t) + c,
\end{align*}
\]

\( U \subseteq \mathbb{R}^m, \quad Y \subseteq \mathbb{R}^p, \) polytopes,

\( X \subseteq \mathbb{R}^n, \) closed full-dim. polytope,

\[ t_1 = \inf\{t \in T \cup \{+\infty\} | x(t) \in F_{n-1,r} \subseteq \partial X\}, \]

**lifetime** of state trajectory;

\[ T_1 = [t_0, \infty), \text{ if } t_1 = \infty, \text{ or,} \]

\[ T_1 = [t_0, t_1] \subseteq \mathbb{R}_+, \text{ if } t_1 < \infty, \text{ then } u, x, y \text{ defined on } T_1, \]

\( F_{n-1,r} \) called **exit facet.**
Discussion of problems for hybrid systems

Problem Control synthesis, realization, and computability of (piecewise-affine) hybrid systems.

Remarks

- Complexity of control synthesis is the main issue for hybrid systems.
- Theory of computation and complexity for discrete sets. (Concept of Turing machine. Decidable and undecidable problems.)
- For real numbers, complexity theory available in books: (1) Blum-Cucker-Shub-Smale. (2) K. Weihrauch (computable analysis). Needed are more concepts, theorems, and experience.
- Problems of reachability and of observability of PAHS are undecidable. (E.D. Sontag; P.J. Collins, JHvS (CDC.2004)).
CWI - Approaches to control, realization, and computability of HS

1. Control synthesis: Computable sufficient conditions for existence of control laws and algorithms for control laws.


3. Computability: For which subclass of nonlinear hybrid system is the reachable subset numerically approximable?
CWI-Approach to control synthesis for PAHS

1. Decomposition into discrete and continuous dynamics.
2. Control at continuous level: affine systems on polytopes.
   (2.1) Control-to-a-facet.
   (2.2) Control-to-exit.
   (2.3) Stabilization-to-a-fixed-point.
3. Control at discrete level: reachability check.
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Concepts for polytopes

Def. A polytope \( P \subset \mathbb{R}^n \) is defined to be a finite intersection of closed half spaces which, moreover, is bounded,

\[
P = \bigcap_{i=1}^{m} \{ x \in \mathbb{R}^n | n_i^T x \leq q_i \},
\]

\[
\dim(P) = \dim(\text{affh}(P)).
\]

(Equivalently, a polytope is the convex hull of a finite number of vectors.)
Representations,

\[
P = \{ x \in \mathbb{R}^n | N^T x \leq q \}, \text{ implicit form;}
\]

\[
= \{ x \in \mathbb{R}^n | \exists y \in S^k_+, x = Ay \}, \text{ explicit form.}
\]
Concepts for polytopes

**Def.** A **simplex** is a polytope for which there exists \( r \in \mathbb{Z}_+ \),

\[
P = \text{convh}(\{v_1, \ldots, v_{r+1}\}) \subset \mathbb{R}^n,
\]

\[
\text{dim}(P) = r.
\]

**Full-dimensional simplex** if \( \text{dim}(P) = n \).

**Def.** **Affine map** \( f : \mathbb{R}^{n_1} \to \mathbb{R}^{n_2} \) if

\[
f(x) = Sx + r, \quad \exists \ S \in \mathbb{R}^{n_2 \times n_1}, \ \exists \ r \in \mathbb{R}^{n_2}.
\]

**Def.** Polytopes \( P_1 \subset \mathbb{R}^{n_1}, P_2 \subset \mathbb{R}^{n_2} \) called **affinely isomorphic** if there exists an affine map,

\[
f : P_1 \to P_2,
\]

which is a bijection between \( P_1 \) and \( P_2 \).
Concepts for polytopes

Def. Consider polytope

\[ P = \{ x \in \mathbb{R}^n | N^T x \leq q \}. \]

Face \( F \) of \( P \) defined as the set,

\[ F = P \cap \{ x \in \mathbb{R}^n | N_s^T x = q_s \}, \]

\[ \dim(F) = \dim(\text{affh}(F)). \]

Facet of \( P \) is face \( F \) such that,

\[ \dim(F) = \dim(P) - 1. \]

Notation

\[ F_{n_{P-1}}(P) = \{ F_{n_{P-1},i} \subset P | \forall i \in \mathbb{Z}_r \}, \text{ set of facets}, \]

\[ F_{n_{P-1},i} = P \cap \{ x \in \mathbb{R}^n | nn_i^T x = q_i \}, \text{ facet } i \in \mathbb{Z}_r = \{1, 2, \ldots r\}. \]

Facet is intersection of polyhedral set with a supporting hyperplane. Lattice of facets fully describes geometric structure of polyhedral set.
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Control-to-facet

**Problem. Control-to-facet.** Consider affine system,

\[
\dot{x}(t) = Ax(t) + Bu(t) + a, \quad x(t_0) = x_0,
\]

\[X = P_n \subset \mathbb{R}^n, \ U \subset \mathbb{R}^m,\] closed polyhedral sets,

\[F_1 \subset P_n, \text{ a particular facet.}\]

**Control objective** Reach specified exit facet. For all \(x_0 \in X\) determine a **terminal time** \(t_1 \in [t_0, \infty)\) and an input \(u : [t_0, t_1] \to U\) such that the state trajectory reaches facet \(F_1:\)

1. \(\forall t \in [t_0, t_1], \ x(t) \in X = P_n;\)
2. \(x(t_1) \in F_1, \ t_1 \in T \text{ smallest such time};\)
3. \(n_1^T \dot{x}(t_1) > 0, \text{ vector field at } F_1 \text{ points outward.}\)

Preferably by continuous control law.
Remark

Consider

\[ \dot{x}(t) = Ax(t) + Bu(t) + a, \quad x(t_0) = x_0, \]

PAHS system,

\[ k(x) = Fx(t) + g, \text{ control law} \]

\[ u(t) = k(x(t)), \text{ input function}, \]

\[ \dot{x}(t) = (A + BF)x(t) + (a + Bg), \quad x(t_0) = x_0, \]

closed-loop system.
Proposition. Necessary conditions.
State set is polytope $X = P_N = \text{convh}(\{v_1, \ldots, v_M\}) \subset \mathbb{R}^N$.

If there exists a continuous control law $f : X \to U$
for Problem ‘Control-to-facet’ with exit facet $F_1$
then there exists $u_1, u_2, \ldots, u_M \in U$ such that

\begin{align*}
(1) \quad \forall j \in V_{F_1} : & \quad (1.a) \quad nn_1^T(Av_j + Bu_j + a) > 0; \\
(1.b) \quad \forall i \in W_{v_j}\{1\} : & \quad nn_i^T(Av_j + Bu_j + a) \leq 0; \\
(2) \quad \forall j \in \mathbb{Z}_M \setminus V_{F_1} : & \quad (2.a) \quad \forall i \in W_{v_j} : \quad nn_i^T(Av_j + Bu_j + a) \leq 0; \\
(2.b) \quad \sum_{i \in W_{v_j}} nn_i^T(Av_j + Bu_j + a) < 0.
\end{align*}

Note, inequalities describe that vector field at the vertices of the polytope of the closed-loop system be in specified polyhedral cone.

Proof outline Define $u_j = f(v_j)$. 
Necessary conditions for control to facet

Vector field of closed-loop system at vertices points in polyhedral cones at vertices.
Theorem Sufficient condition - Simplex case.
Case $X = S_n \subset \mathbb{R}^n$ full-dimensional simplex.

$$\dot{x}(t) = Ax(t) + Bu(t) + a, \ x(t_0) = x_0.$$ 

Assume necessary conditions (1.a,1.b,2.a,2.b) hold for $u_1, \ldots, u_{n+1} \in U$. Define

$$f(x) = \sum_{j=1}^{n+1} \lambda_i u_i, \text{ if } x = \sum_{j=1}^{n+1} \lambda_i v_i,$$

$$f : S_n \rightarrow U, \text{ affine control law.}$$

Then $f$ is a solution to Problem ‘Control-to-facet’ for exit facet $F_1$.

Remark Conditions (1.a,1.b,2.a,2.b) interpreted as controllability conditions for the control problem control-to-facet. Geometric approach to control of affine systems on simplices.
Figure: Control of the vector field $\dot{x}$ at the vertices of $S_2$
Proof outline

- Conditions imply that at vertices the vector field points in the objective directions.
- Because of convexity, on exit facet, vector field points out off polytope, on other facets, vector field points to inside of polytope.
- From any point in polytope, exit facet is reached in finite time.
Solution procedure

1. Solve set of linear inequalities (1.a,1.b,2.a,2.b) for $u_1, \ldots, u_{n+1} \in U$.
   Software for linear inequalities:
   - CDD Library (Fukuda (ETHZ)).
   - New Polka Library (B. Jeannet (Verimag)).

2. Solve the following linear equation for $F \in \mathbb{R}^{m \times N}, g \in \mathbb{R}^m$,
   \[
   \begin{pmatrix}
   v_1^T & 1 \\
   \vdots & \vdots \\
   v_{n+1}^T & 1 \\
   \end{pmatrix}
   \begin{pmatrix}
   F^T \\
   g^T \\
   \end{pmatrix}
   =
   \begin{pmatrix}
   u_1^T \\
   \vdots \\
   u_{n+1}^T \\
   \end{pmatrix}.
   \]
   Matrix with $v$’s is nonsingular because simplex is full-dimensional.

3. Control law is $u = Fx + g$. 
Control-to-facet

Theorem. Sufficient conditions - Polytope case.

Case $X = P_N$ full-dimensional polytope. Consider FDAPS. Assume there exist $u_1, \ldots, u_M \in U$ satisfying (1.a - 2.b) and

$$(2.c) \quad \forall j \in Z_M \setminus V_{F_1} : nn_1^T (Av_j + Bu_j + a) > 0.$$ 

(a) There exists a continuous and piecewise affine map

$$\xi : X = P_N \rightarrow T_M, \quad x = \sum_{j=1}^{M} \xi(x)_j v_j \in P_N;$$

(b) Define

$$\psi : T_M \rightarrow U, \quad \psi(\lambda) = \sum_{j=1}^{M} \lambda_j u_j, \quad f : P_N \rightarrow U, \quad f = \psi \circ \xi.$$ 

Then $f$ is a continuous piecewise affine control law and a solution to Problem ‘Control-to-facet’ for exit facet $F_1$. 
Solution procedure

Procedure

1. Partition polytope in simplices such that,

\[ \{ \text{vertices simplices} \} \subseteq \{ \text{vertices polytope} \}. \]

Algorithms [C.W. Lee, 1997].

2. For each simplex an affine control law.

3. Overall control law piecewise-affine and continuous.

Remarks

1. Case multi-dimensional rectangles. Often occurs in applications. Necessary and sufficient differ only in terms of \(<\) and \(\leq\).

2. For online computation only the simplices needed through which trajectory travels.
Reachability and control synthesis for PAHS

Problem Control synthesis for PAHS on simplex

\[ \dot{x}_q(t) = A(q)x_q(t) + B(q)u(t) + a(q), \quad x_q(t_0) = x_{q,0}, \]

\( Q \) finite set, \( U \subset \mathbb{R}^m \) polytope,

\( X_q \subset \mathbb{R}^{n_q} \) simplex, \( G_q(e) \subset \partial X_q \) guards contained in facets,

\( Q_u \subset Q \) unsafe locations,

\( Q_s \subset Q \setminus Q_u \) start locations, \( q_t \in Q \setminus Q_u \) target location.

Control objective Reach terminal state (either discrete state only or discrete and continuous state) Determine control laws,

\[ k_q(x) = F_q x + g_q, \quad k_q : X_q \to U, \quad \forall q \in Q, \]

such that \( \exists t_1 \in [t_0, \infty) \) and

\( (t_0, q_s, x_{q_s,s}) \in T \times Q_s \times X_{q_s} \leftrightarrow (t_1, q_t, x_{q_t,t}) \in T \times Q \times X_{q_t} \)

either stay at target location or converge to fixed point \( x_{q_t,t} \in X_{q_t}. \)

Remarks (1) Sufficient condition; (2) Computationally tractable.
Approaches to control problem for affine system on polytope

1. Determine a control law such that the state trajectory of the closed-loop system can leave through a facet of the polytope.

2. Determine a control law such that the state trajectory of the closed-loop system leaves the simplex in finite time through one or more of prespecified facets.

3. For a target state $x_t \in X$ determine a control law such that

$$\lim_{t \to \infty} x_{q_i}(t; (t_2, x_{q_i,2})) = x_t,$$

and never leaves the target location.
Problem Control-to-a-facet

\[ \dot{x}(t) = Ax(t) + Bu(t) + a, \quad x(t_0) = x_0, \]
\[ X \subset \mathbb{R}^n \text{ simplex, } U \subset \mathbb{R}^m \text{ polytope,} \]
\[ I \subseteq \mathbb{Z}_{N+1} = \{1, \ldots, N+1\}, \]
\[ \{F_i \subset \partial X, i \in I\} \text{ admissible exit facets.} \]

Determine an affine control law,

\[ k(x) = Fx + g, \]
\[ \dot{x}(t) = (A + BF)x(t) + (a + Bg), \quad x(t_0) = x_0, \text{ closed-loop system,} \]

such that,

if the state trajectory of the closed-loop system leaves the simplex
then it does so through one of the admissible exit facets.
Theorem **Control-to-a-facet**
There exists an affine control law for this problem if and only if

\[ \exists u_1, \ldots, u_{N+1} \in U \text{ such that,} \]
\[ n_i^T (Av_j + Bu_j + a) \leq 0, \quad \forall i \in \mathbb{Z}_{N+1} \setminus I, \quad \forall j \in \mathbb{Z}_{N+1} \setminus \{i\}. \]

Then \((F, g)\) are the unique solution of the equation,

\[
\begin{pmatrix}
    v_1^T & 1 \\
    \vdots & \vdots \\
    v_m^T & 1
\end{pmatrix}
\begin{pmatrix}
    F^T \\
    g^T
\end{pmatrix}
= \begin{pmatrix}
    u_1^T \\
    \vdots \\
    u_{N+1}^T
\end{pmatrix}.
\]

**Remarks** Linear inequalities solvable by computer programs.
**Problem Control-to-exit.** Consider,

\[
\dot{x}(t) = Ax(t) + Bu(t) + a, \quad x(t_0) = x_0,
\]

\(X \subset \mathbb{R}^N\) simplex, \(I \subset \mathbb{Z}_{N+1}\),

\(\{F_i \subset \partial X, i \in I\}\) admissible exit facets.

Determine an affine control law

\[
k(x) = Fx + g,
\]

such that the state trajectory of the closed-loop system leaves the polytope \(X\) via an admissible exit facet in finite time.
Theorem Control-to-exit

\[ U_j = \left\{ u \in U | n_i^T (A v_j + B u + a) \leq 0, \forall i \in \mathbb{Z}_{N+1} \setminus (I \cup \{j\}) \right\}, \]

\[ W_j = \text{vertices of } U_j, \quad \forall j \in \mathbb{Z}_{N+1}. \]

The problem is solvable if and only if

\[ \forall j \in \mathbb{Z}_{N+1} \exists w_j \in W_j \text{ such that}, \]

\[ 0 \not\in \text{convh}(\{ A v_j + B w_j + a | \forall j \in \mathbb{Z}_{N+1} \}). \]

Remark Linear equalities to be checked for condition.
**Theorem Existence of fixed points**
Consider an autonomous affine system on a polytope

\[ \dot{x}(t) = Ax(t) + a, \quad x(t_0) = x_0, \quad X \subset \mathbb{R}^N. \]

There exists a **fixed point**,

\[ 0 = Ax_f + a, \quad x_f \in X, \]

if and only if

\[ \exists x_0 \in X \text{ such that } \forall t \in [t_0, \infty), \quad x(t; t_0, x_0) \in X. \]
Problem Stabilization to a fixed point

Consider the affine system on a simplex,

\[ \dot{x}(t) = Ax(t) + Bu(t) + a, \quad x(t_0) = x_0, \]

\[ X \subset \mathbb{R}^N \text{ simplex}, \quad U \subset \mathbb{R}^m \text{ polytope}, \quad x_f \in X \text{ fixed point}. \]

Determine an affine control law,

\[ k(x) = Fx + g, \]

such that,

1. control law \( k \) is admissible: \( \forall x \in X, \ k(x) \in U; \)
2. state trajectory is admissible: \( \forall t \in T, \ x(t; t_0, x_0) \in X; \)
3. state trajectory converges to fixed point: \( \lim_{t \to \infty} x(t; t_0, x_0) = x_f. \)
Theorem Stabilization-to-a-fixed-point

Problem is solvable if and only if

\[ \exists u_1, \ldots, u_{N+1} \in U, \text{ such that} \]

\[ (1) \quad n_i^T (Av_j + Bu_j + a) \leq 0, \quad \forall j \in \mathbb{Z}_{N+1}, \quad \forall i \in \mathbb{Z}_{N+1} \setminus \{j\}; \]

\[ (2) \quad B \sum_{j=1}^{N+1} \mu_j v_j = -Ax_f - a; \]

\[ (3) \quad \text{span}(\{Av_j + Bu_j + a|\forall j \in \mathbb{Z}_{N+1}\}) = \mathbb{R}^N. \]

Notation \( x_f = \sum_{j=1}^{N+1} \mu_j v_j. \)
Def. Discrete-event system

\[ DES = (Q, E, f), \quad Q \text{ state set, } E \text{ event set,} \]
\[ f : \text{Dom}(f) \subset (Q \times E) \rightarrow Q \text{ transition function,} \]
\[ E_q = \{ e \in E | (q, e) \in \text{Dom}(f) \}, \quad \text{subset of eligible events, } \forall q \in Q, \]
\[ (q_0, q_1, \ldots, q_n), \quad q_i = f(q_{i-1}, e_i). \]

For control law \( k_q \) define,

\[ (s_q, m_q) \quad \text{local supervisor}, \quad s_q \in E_q, \quad m_q \in \{ \bot, \top \}, \quad \forall q \in Q, \]
\[ e \in s_q \text{ if } x_q(t) \text{ leaves } X_q \text{ through guard } G_q(e); \]
\[ m_q = \top, \quad \text{if there exists a fixed point } x_{q,f} \in X_q. \]
Problem Reach-avoid problem for a PAHS.

\[
DES = (Q, E, f),
\]

\[Q_s \subset Q \text{ starting states, } q_t \in Q \text{ target state, } Q_u \in Q \text{ unsafe states}\]

Determine a supervisor \( S \) such that,

1. \( S/DES \) is nonblocking;
2. \( q_0 \in Q_s \) and there exists an integer \( n \in \mathbb{Z}_+ \) such that \( q_n = q_t \),
   determine a path; minimize cost associated with transitions;
3. \( \forall i \in \mathbb{N}_n, q_i \notin Q_u \).
4. **Reach-avoid-stabilize**, in addition,
   \( s_{q_t} = \emptyset \) and \( m_{q_t} = \top \).
5. **Reach-avoid-converge**, in addition,
   \( \lim_{t \to \infty} x_{q_t}(t) = x_f \).
Algorithm **Reach-avoid problem** (like dynamic programming)

1. If \( q_t \in Q_u \) then terminate.
2. \( Q_0 = \{ q_t \} \) and choose a set of supervisors \( S_{q_t} \) and \( i = 0 \).
3. While not \( (Q_s \subset Q_j \text{ or } Q_j = Q_{j-1}) \) do \( i = i + 1 \),

\[
Q_i = Q_{i-1} \cup \\
\cup \left\{ q \in Q \setminus Q_u | \exists (s_q, m_q) \in S_q \text{ such that } f(q, s_q) \subset Q_{i-1} \right\} ,
\]

\[
S_q \text{ local supervisor found in (3.1)}
\]

Output: \( Q_j \) and \( \{ s_q \in S_q, q \in Q_j \} \).

**Theorem 6.3 Reach-avoid problem**
If Algorithm 6.2 terminates with \( Q_s \subset Q_j \)
then there exists a solution to Problem 6.1.
Example Reach-avoid-stabilize

\[ \dot{x}_q(t) = A(q)x_q(t) + B(q)u(t) + a(q), \quad x_q(t_0) = x_0, \]
\[ Q = \{ q_1, \ldots, q_5 \}, \quad E = \{ e_1, \ldots, e_5 \}, \]
\[ X_q = \{ x \in \mathbb{R}^2 | x_1 \geq 0, x_2 \geq 0, x_1 + x_2 \leq 1 \}, \quad \forall q \in Q, \]
\[ G_{q_1}(e_2) = F_3, \quad G_{q_1}(e_3) = F_2, \quad G_{q_1}(e_4) = F_1, \quad \text{etc.} \]
\[ \dot{x}_{q_1} = \begin{pmatrix} 3 & 0 \\ -1 & 2 \end{pmatrix} x_{q_1}(t) + \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \text{etc.} \]
\[ Q_s = \{ q_1 \}, \quad q_t = q_5, \quad Q_u = \{ q_4 \}. \]

(Continued on next slide)
Example (Continued)
Control law specified by:

\[ k_{q_1}(x) = 1, \quad q_1 \leftrightarrow q_2, \]
\[ k_{q_2}(x) = 0, \quad q_2 \leftrightarrow q_3, q_5, \]
\[ k_{q_3}(x) = 1, \quad q_3 \leftrightarrow q_5, \]
\[ k_{q_5}(x) = -x_1 - \frac{3}{4} x_2 + \frac{1}{2}, \quad q_5 \leftrightarrow q_5. \]

Diagrams of discrete-event systems, see next slide.

Example Two-tank systems.

Tool ConPAHS.
Diagram of discrete-event system
(a) Open-loop system; (b) Closed-loop system.
Further results on control and realization of PAHS

- Control of PAHS on polytopes with one affine control law per polytope. Previous case was for PAHS on simplices. Sufficient condition for control synthesis with low complexity of controller.

- Computer program package **ConPAHS** (Anton Kuut, Margreet Nool).

- Realization of hybrid systems (Mihály Petreczky).
Research in control of hybrid systems

Motivation

- Control of engineering systems with computers.
- Distributed hybrid systems (networks).

Theory for control of hybrid systems

- Complexity of control synthesis.
- Complexity of online computation of inputs.
- Approach 1. Selection of class of systems. PAHS. System approximation, system reduction.
- Approach 2. Approximation in control synthesis and in control design.
- Control of distributed hybrid systems. Coordination at global level.
Applied research for control of hybrid systems

- Tool development. ConPAHS.
- Examples of control engineering problems.
- Examples of control of distributed systems.
Concluding remarks

**Results** Control of PAHS:
2. Control-to-exit.
4. Control synthesis for PAHS.

**Research plan**
1. Control with partial observations of PAHS.
2. Computer program package ConPAHS.
3. Realization of PAHS on polytopes.
4. Computability of hybrid systems.
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The end!