0.1 Detumbling controller based on B-dot

The first and most important attitude control task to be executed after orbital insertion of a satellite is stabilizing its anglular rate, i.e. detumbling. This procedure should be done by a robust and failsafe system which does not depend on "very complex" systems being operational like e.g. attitude estimation filters. A very simple solution to detumbling using magnetic actuation is the $\dot{B}$ (B-dot) algorithm.

The principle of a $\dot{B}$-controller is to minimize the derivative of the magnetic field vector measured by a magnetometer. As the spacecraft orbits the earth, the magnetic field vector in the spacecraft reference frame changes depending on the position of the spacecraft. However, the dominant rate of change in direction of the field vector is caused by the tumbling of the satellite as it may tumble with angular rates much larger than the orbital rate. Minimizing the change in the measured field vector by means of actuation causes the spacecraft to approach an angular rate close to the orbital rate which is acheived by forcing the derivative of the measured B-field, $\dot{B}$, to zero.

0.1.1 B-dot control law

The control law for $\dot{B}$ is nice and neat and can be written as equation 1.

$$m_{mt} = -C\,{}^{S}\dot{B} \tag{1}$$

Where m_{mt} is the magnetic dipole moment vector to be generated by the magnetic actuators in the three axes of the spacecraft. C is a controller gain and $\dot{B}$ is the time derivative of the magnetic field vector. The controller gain is negative in order to actuate opposite the rotation thus taking kinetic energy out of the system.

The reason that $\dot{B}$ can be used directly without any cross product is that the changes in the B-field that the controller seeks to minimize are caused by a rotation of the spacecraft and hence the derivative of the B-field is perpendicular to the field vector. This means that the control law gives an output to the actutors which is a dipole moment perpendicular to the B-field.

$\dot{B}$ can be written as equation (2).

$${}^{S}\dot{B} \approx {}^{S}B \times {}^{O}\omega_{sc} \tag{2}$$

given the assumption that the direction and magnitude of the B-field with respect to the orbit fixed coordinate system, ${}^{O}B$, is constant. This assumption leads to the conclusion that the rate of change of the B-field in the spacecraft refence frame is mainly due to the rotation of the spacecraft.

0.1.2 Estimating B-dot

One problem with the $\dot{B}$-algorithm is that ${}^{S}\dot{B}$ cannot be directly measured by the magnetometer and differenting its output may give peaks of unwanted noise.

Continuous time estimation

A continuous time filter that estimates the rate of change of the B-vektor can be realized and simply multiplying its output by the controller gain C gives the controller output to the magnetorquers as a dipole moment reference. See figure 1.

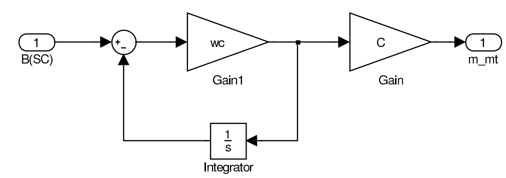

Figure 1: *Filter for estimating the time derivative of the B-field.*

The transferfunction of the filter, disregarding C, is given in equation 3.

$$H_{cont}(s) = \frac{{}^{S}\hat{\dot{B}}}{{}^{S}B} = \frac{\omega_c s}{s+\omega_c} \tag{3}$$

Discrete time estimation

In reality, the controller will be implemented on a computer and therefor needs to be discrete. Using pole-zero matching and gain matching the transferfunction of the $\dot{B}$-estimator can be written in the z-domain as shown in equation (4).

$$p_{cont} = -\omega_c \qquad p_{disc} = e^{-\omega_c T_s}$$
$$z_{cont} = 0 \qquad z_{disc} = 1$$

$$H_{disc}(z) = K\frac{z-1}{z-e^{-\omega_c T_s}} \tag{4}$$

The gain is matched in center of the bandwith, that is at $\omega_0 = \frac{\omega_c}{2}$ and the gain correction K is computed as follows:

$$K = \frac{K_{cont}}{K_{disc}} = \frac{\|H_{cont}(s)\|}{\|H_{disc}(z)\|}\Bigg|_{s=j\omega_0, z=e^{-j\omega_0 T_s}} \tag{5}$$

0.1.3 Periodic measurement and actuation

The control law described in section 0.1.1 is stated without any constraints to the measured B-field or the output to the magnetic actuators. However, using the actuators while trying to measure the B-field with the magnetometer causes a disturbance to the measurements that introduces a feedback in the control loop which cannot easily be estimated. In order to dodge this potential problem the actuators and the sensor are not used simultaniously but a periodic time-sharing policy is adopted.

The period of the control/measurement cycle is $T_{cycle} = T_{sensor} + T_{actuator}$. During the period T_{sensor} the sensor readings are fed to the discrete $\dot{B}$-estimation filter which settles to an estimate of the rate of change of the B-field. During the rest of the time of the cycle period, $T_{actuator}$, the output from the controller is held at a constant value yielding a constant magnetic dipole moment from the actuators. All readings from the magnetometer are discarded in the actuation period and the input to the estimation filter is held at zero. Figure 2 illustrates the principle.

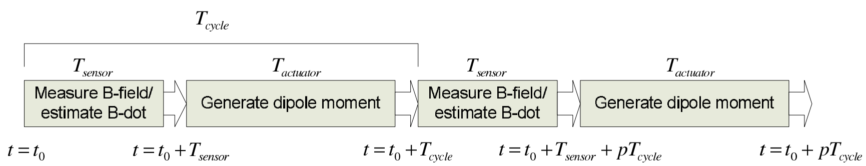

Figure 2: *Periodic measurement and actuation.*

0.1.4 Stability of the B-dot controller

In order to analyze stability of the detumbling controller the ideal[1] continuous time $\dot{B}$-controller is first considered.

Lyapunov stability

The stability using the control law in (1) can be proven by the Lyapunov direct method described in chapter 5 in [rafal]. Since the stability criterium for detumbling implies that the rotational kinetic energy of the satellite should converge to zero[2], a Lyapunov candidate function is equation (6).

$$E_{kin} = \frac{1}{2}\, {}^{o}\omega_{sc}{}^{T} I_{sc}\, {}^{o}\omega_{sc} \tag{6}$$

In order to ensure energy dissipation and thus stability the derivative of the kinetic energy must be negative definite. Neglegting external disturbances the change in kinetic energy is due only

[1]By “ideal” means the assumption that the excat time derivative of the B-field can be measured.
[2]This is an assumption as the orbital rate of the satellite is not considered.

to the torque excerted by the magnetorquers. Hence, the change in energy is

$$\dot{E}_{kin} = {}^{o}\omega_{sc}{}^{T}\, {}^{s}N_{mt} \tag{7}$$

Including the control law into equation (7) and using (2) a simple expression for the change in kinetic energy can be acheived:

$$\begin{aligned}
\dot{E}_{kin} &= {}^{o}\omega_{sc}{}^{T} (-C\, {}^{s}\dot{B} \times {}^{s}B) \\
&= -C\, {}^{o}\omega_{sc}{}^{T} ({}^{s}\dot{B} \times {}^{s}B) \\
&= -C\, {}^{o}\omega_{sc}{}^{T} (S({}^{s}\dot{B})\, {}^{s}B) \\
&= C\, {}^{o}\omega_{sc}{}^{T} (S({}^{s}B)\, {}^{s}\dot{B}) \\
&= C (S({}^{s}B)\, {}^{s}\dot{B})^{T}\, {}^{o}\omega_{sc} \\
&= C\, {}^{s}\dot{B}^{T} (S({}^{s}B)^{T}\, {}^{o}\omega_{sc}) \\
&= -C\, {}^{s}\dot{B}^{T} (S({}^{s}B)\, {}^{o}\omega_{sc}) \\
&= -C\, {}^{s}\dot{B}^{T} ({}^{s}B \times {}^{o}\omega_{sc}) \\
&= -C\, {}^{s}\dot{B}^{T}\, {}^{s}\dot{B} \\
&= -C \| {}^{s}\dot{B} \|^{2}
\end{aligned} \tag{8}$$

Equation (8)[3] describes the change of rotational kinetic energy of the spacecraft when applying the $\dot{B}$ control law. This equation is negative definite thus proving that energy is dissipated from the system during detumbling. The control gain C determines the rate of energy dissipation and can be selected according to the detumbling requirements and electrical power constraints etc. The result of the Lyapunov analysis also shows that energy dissipation in the detumbling phase is proportional to $\|\dot{B}\|^2$ which means that angular rates are reduced rapidly after initiating B-dot control and slowly converging over time.

Stability including the B-dot estimation filter

The implemented controller is unfortunately not an ideal $\dot{B}$ controller and the Lyapunov stability analysis suggested in [rafal] must be used with some modifications/extensions. The result in (8) can be adopted to include the estimated derivavtive of the B-field as follows:

$$\dot{E}_{kin} = -C\, {}^{s}\hat{\dot{B}}^{T}\, {}^{s}\dot{B} \tag{9}$$

In order for (9) to stay negative definite the vector dot-product ${}^{s}\hat{\dot{B}}^{T}\, {}^{s}\dot{B}$ must be negative at all times which can only be ensured if the absolute angle between ${}^{s}\hat{\dot{B}}$ and ${}^{s}\dot{B}$ is less that 90°. If the angle is more than 90° then ${}^{s}\hat{\dot{B}}^{T}\, {}^{s}\dot{B}$ becomes negative and (9) becomes postive and the kinetic energy rises. In this case the Lyapunov analysis does not prove stability.

[3]$S(B)$ is the skew symmetric cross product matrix that is used to reduce cross products to matrix multiplications. The proporties of skew symmetric matrices have been used to manipulate the equation.

Figure 3 illustrates the problem.

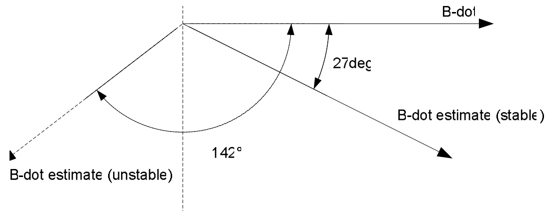

Figure 3: *The B-dot vector and the estimated B-dot vector used in the Lyapunov stability analysis. The absolute angle between thees vectors must be less than* 90° *in order for the stability analysis to be valid.*

By rewriting (3) the relationship between ${}^{s}\dot{B}$ and ${}^{s}\hat{\dot{B}}$ is found:

$$\frac{{}^{s}\hat{\dot{B}}}{{}^{s}\dot{B}\frac{1}{s}} = \frac{\omega_c s}{s+\omega_c} \tag{10}$$

$$\frac{{}^{s}\hat{\dot{B}}}{{}^{s}\dot{B}} = \frac{\omega_c}{s+\omega_c} \tag{11}$$

This transferfunction has a phase in the interval $[0^\circ; -90^\circ[$ and a phase of -45° at the bandwidth frequency. The interpretation of this is that when the rate of change of $\dot{B}$ is large the angle between ${}^{s}\dot{B}$ and ${}^{s}\hat{\dot{B}}$ increases and stability becomes marginal; i.e. the phase margin decreases.

Phase of estimated B-dot The phase of (11) is dependant on the rate of change of $\dot{B}$, that is the size of $\ddot{B}$. Equation (12) expresses $\ddot{B}$ with the assumption there are no spinning momentum wheels in the spacecraft. Also, it is assumed that the geomagnetic field is constant in the inerital coordinatesystem to start with in order to simplify the equations.

$$\begin{aligned}
{}^{s}\dot{B} &= {}^{s}B \times {}^{s}\omega_{sc} \\
{}^{s}\ddot{B} &= {}^{s}\dot{B} \times {}^{s}\omega_{sc} + {}^{s}B \times {}^{s}\dot{\omega}_{sc} \\
{}^{s}\dot{\omega}_{sc} &= I^{-1}({}^{s}N_{ctrl} - {}^{s}\omega_{sc} \times I\, {}^{s}\omega_{sc}) \\
{}^{s}\ddot{B} &= {}^{s}\dot{B} \times {}^{s}\omega_{sc} + {}^{s}B \times (I^{-1}({}^{s}N_{ctrl} - {}^{s}\omega_{sc} \times I\, {}^{s}\omega_{sc}))
\end{aligned} \tag{12}$$

It is clear that the phase increases with the actuation torque which is propotional to the controller gain C and the size of $\dot{B}$ and inverse propotional to the inertia of the satellite. Also, there will be a contribution to the rate of $\dot{B}$ from the local variations in the B-field caused by the change of position of the spacecraft (i.e. its position in orbit). Measurement noise and pure time delay from the magnetometer may add even more phase to the estimated $\dot{B}$.

A simlutaion has been made with some pre-selected values of ω_c and C which are based on the suggested values from [acs-cubesat] in which a similar control problem is handled. Setting $\omega_c = 0.7$ and $C = -11000$ the control law including the $\dot{B}$ yields the behavior illustrated in the simulation results of figure 4 and 5. The satellite is detumbling from an initial angular velocity of $[0.2 0.2 0.2]\,\frac{\text{rad}}{\text{s}}$.

Figure 4 shows the norm of the angular velocity of the satellite during the simulation. The angluar velocity norm is monotonously decreasing as predicted by the Lyapunov analysis, which seems resonable when observing the phase of the estimated $\dot{B}$ in figure 5.

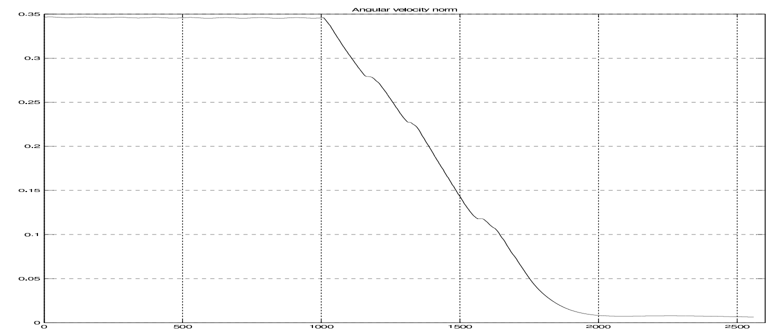

Figure 4: *Result of a simulation with the continuous time B-dot controller enabled after t=1000 seconds. The graph shows the norm of the angular velocity of the satellite while detumbling.*

Figure 5 shows that the phase of the estimated derivative of the B-field is approximately in the interval $[0^\circ; 20^\circ]$ which gives a margin of 70° to the dangerous 90° that may cause instability.

Stability using periodic measurement and actuation

As described in section 0.1.3 the derivative of the B-field cannot be estimated during the actuation period due to magnetic disturbances from the magnetorquers. This means that the $\hat{\dot{B}}$ estimate will be constant for the period of actuation. As torque is applied when actuating the actual $\dot{B}$ will change thus creating a difference between $\dot{B}$ and $\hat{\dot{B}}$ which is equivalent to a change in the angle between these. This angle must be added to the phase of the $\hat{\dot{B}}$-estimation filter when considering stability.

According to figure 5 the maximum additional angle allowed is approximately 70°. The figure shows that the period of $\dot{B}$ is at least $20\,\text{s}$. Hence, the maximum time allowed for the controller to use a constant estimate of $\dot{B}$ is $\frac{70^\circ}{360^\circ}20\,\text{s} = 3.8\,\text{s}$ which potentially (worst case) adds 70° to the anlge between $\dot{B}$ and $\hat{\dot{B}}$. There are no requirements to the time of measuring the B-field between actuation regarding stability of the system. However, the measurement time should be long enough for the filter to settle to an acceptable estimate of $\dot{B}$, otherwise the error introduced here will also contribute to the phase of $\hat{\dot{B}}$.

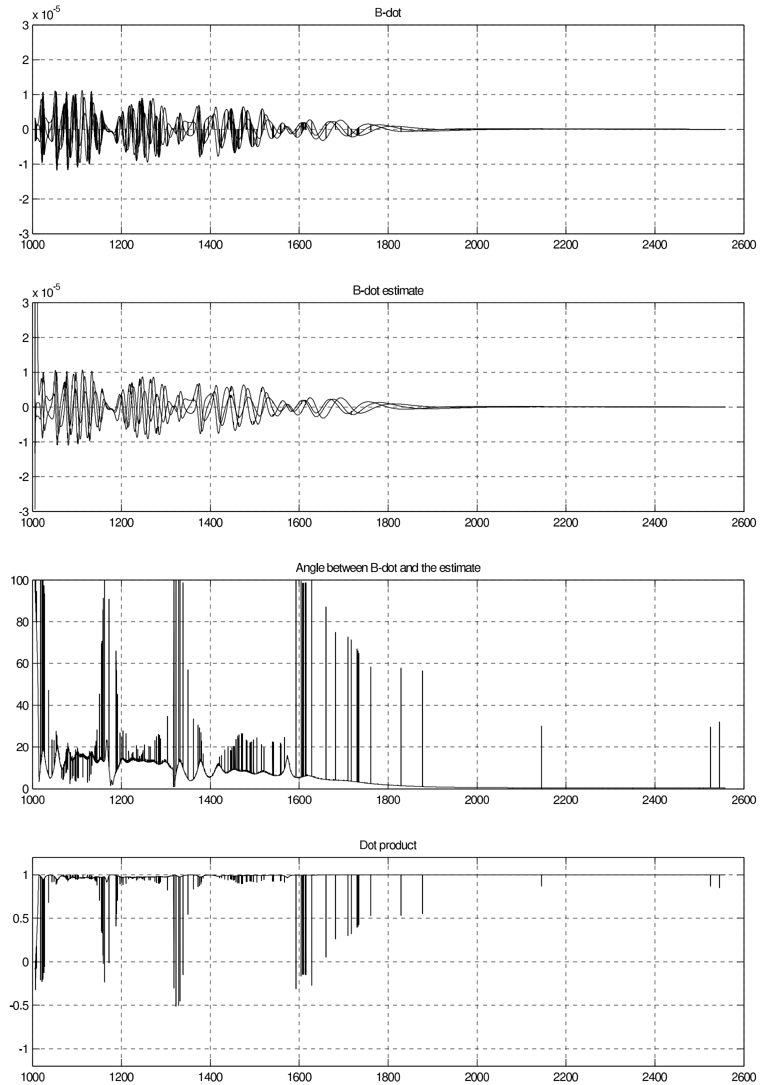

Figure 5: *Result of a simulation with the continuous time B-dot controller enabled after t=1000 seconds. First graph is the real derivaties of the B-field. Second graph is the estimated derivative. Third graph is the angle between B-dot and the estimated B-dot. Last graph is the dotproduct of the two first. The peaks in the graphs are caused by bugs in the Rømer IGRF simulation software.*

0.1.5 Implementation

The controller is implemented in simulink to test it together with the AAUSAT-II simulation library.

0.1.6 Test

Coming soon.